Pop Promo Packet

The promo packet contains literacy for basic problems and the new differentiation and limit mean. The new differentiation only removes one concept to increase the derivative power to have motion. The new limit mean is identified in the integral and removed for thousands of more theorems. The book contains a collection of every theorem that has been fixed with the limit mean.

Table of Contents

Chapter 1: Basics..........1
  Section 1.1: Arithmetic and division..........1
  Section 1.4: Proper bracket notation for numbers..........2
  Section 1.7: Proper bracket notation for counter and addition operators..........2
Chapter 3: Mathematical literacy basics..........5
  Section 3.1: Addition operator..........5
  Section 3.2: Counter operator and addition operator theorems..........5
  Section 3.3: Addition and counter operators for surface functions..........8
  Section 3.4: Repeatable domains..........10
  Section 3.5: Reciprocating functions..........10
Chapter 7: New differentiation..........11
  Section 7.1: Horizontal derivative..........11
  Section 7.2: Assigning a derivative..........12
  Section 7.3: Range derivative..........12
Chapter 8: The mean..........14
  Section 8.1: The mean..........14
  Section 8.2: Mean experiment for the base function..........18
  Section 8.3: The mean of trigonometric functions..........20
  Section 8.4: The mean of a polynomial..........21
  Section 8.5: Convergence for means..........22
Chapter 9: Integration for area and volume..........24
  Section 9.1: Integration..........24

Chapter 1: Basics

Section 1.1: Arithmetic and division

Theorem 1: Arithmetic without a variable has a function f.

Theorem 2: A function has four types of arithmetic addition, subtraction, multiplication and division.

Definition 1: Two numbers a and b have a sum c with an addition operator symbol.

$$f = a + b$$

$$f = c$$

Definition 2: A number a that is equal to the sum c minus b has a subtraction operator symbol.

$$a = c - b$$

Definition 3: A number a that has a count of n has a multiplication operator symbol for the sum of c.

$$f = a + a + \cdots + a$$

$$f = n \cdot a$$

The multiplication operator does not have to be written.

$$f = na$$

$$f = c$$

Section 1.4: Proper bracket notation for numbers

Theorem 1: A variable x has a single bracket with no lower base that is an open point and closed point line segment. Point

$$x\rceil_a^b = (a, b]$$

Theorem 2: A variable x has a single bracket with no top base that is a closed point and open point line segment.

$$x\rfloor_a^b = [a, b)$$

Theorem 3: A variable x has a single bracket with no bases that is a completely open point line segment.

$$x|_a^b = (a, b)$$

Theorem 4: A variable x has a pair of brackets for the difference in two line segments.

$$[x]_a^b = b - a$$

Section 1.7: Proper bracket notation for counter and addition operators

Theorem 1: A function $f(x)$ has open and closed bounds for a counter operator of terms $g(x)$ that has open and closed bounds.

Theorem 2: A set of points or a list has curly brackets for a closed set.

Definition 1: A function $f(x)$ that has a counter operator of two closed bounds has a set of points from a to b that are open points.

$$f(x)]_{a}^{z} = \left. \dot{\iota}\, g(x) \right]_{a}^{z} = \mathop{\dot{\iota}}\limits_{x=a}^{x=z} g(x) = \{g(a), \dots, g(z)\}$$

Definition 2: A function $f(x)$ that has a counter operator of an open bound and a closed bound has a set of open points that has one and one curly bracket.

$$f(x)\rceil_{a}^{z} = \left. \dot{\iota}\, g(x) \right\rceil_{a}^{z} = \mathop{\dot{\iota}}\limits_{x>a}^{x=z} g(x) = (g(a), \dots, g(z)\}$$

Definition 3: A function $f(x)$ that has a counter operator of a closed bound and an open bound has a set of open points that has one closed bound and one open bound.

$$f(x)\rfloor_{a}^{z} = \left. \dot{\iota}\, g(x) \right\rfloor_{a}^{z} = \mathop{\dot{\iota}}\limits_{x=a}^{x<z} g(x) = \{g(a), \dots, g(z))$$

Definition 4: A function $f(x)$ that has a counter operator of two open bounds has a set of points that has two open bounds.

$$f(x)|_{a}^{z} = \left. \dot{\iota}\, g(x) \right|_{a}^{z} = \mathop{\dot{\iota}}\limits_{x>a}^{x<z} g(x) = (g(a), \dots, g(z))$$

Theorem 3: A function $f(x)$ has a continuous counter operator that has open or closed bounds for inequalities.

Definition 5: A function $f(x)$ has a continuous counter operator of two closed bounds has an open point line domain that is closed.

$$f(x)]_{a}^{z} = \left. \dot{\iota}_{c}\, g(x) \right]_{a}^{z} = \mathop{\dot{\iota}_{c}}\limits_{x=a}^{x=z} g(x) = [g(a), g(b)]$$

The closed brackets are for the inequality and all points are still open.

$$g(a) \leq g(x) \leq g(b)$$

Definition 6: A function $f(x)$ has a continuous counter operator of an open bound and a closed bound for an open and closed range.

$$f(x)\rceil_{a}^{b} = \left. \dot{\iota}_{c}\, g(x) \right\rceil_{a}^{b} = \mathop{\dot{\iota}_{c}}\limits_{x>a}^{x=b} g(x) = (g(a), g(b)]$$

Definition 7: A function $f(x)$ has a continuous counter operator of a closed bound and an open bound for an open and closed range.

$$f(x)\rfloor_a^b = i_c\, g(x) \Bigg\rfloor_a^b = \underset{x=a}{\overset{x<b}{i_c}}\ g(x) = \big[g(a), g(b)\big)$$

Definition 8: A function $f(x)$ has a continuous counter operator of two open bounds has an open bounded range.

$$f(x)|_a^z = i_c\, g(x) \Bigg|_a^z = \underset{x>a}{\overset{x<z}{i_c}}\ g(x) = \big(g(a), g(b)\big)$$

Theorem 4: A function $f(x)$ has open and closed bounds for an addition operator of terms $g(x)$ that has open and closed bounds.

Definition 9: A function $f(x)$ has two closed bounds for an addition operator that has two closed bounds.

$$f(x)]_a^b = \mathsf{A}\, g(x) \Bigg]_a^b = \underset{x=a}{\overset{x=b}{\mathsf{A}}}\ g(x)$$

Definition 10: A function $f(x)$ has one open bound and one closed bound for an addition operator that has one open bound and one closed bound.

$$f(x)\rceil_a^b = \mathsf{A}\, g(x) \Bigg\rceil_a^b = \underset{a<x}{\overset{x=b}{\mathsf{A}}}\ g(x)$$

Definition 11: A function $f(x)$ has one closed bound and one open bound for an addition operator that has one closed bound and one open bound.

$$f(x)\rfloor_a^b = \mathsf{A}\, g(x) \Bigg\rfloor_a^b = \underset{x=a}{\overset{x<b}{\mathsf{A}}}\ g(x)$$

Definition 12: A function $f(x)$ has two open bounds for an addition operator that has two open bounds.

$$f(x)|_a^b = \mathsf{A}\, g(x) \Bigg|_a^b = \underset{a<x}{\overset{x<b}{\mathsf{A}}}\ g(x)$$

Chapter 3: Mathematical literacy basics

This chapter increases mathematical literacy for basic problems and has long division and number theory.

Section 3.1: Addition operator

The addition operator is the operator before the summation and the product.

Theorem 1: A function is a line from $a \leq x \leq b$ only.

$$f(x) = \underset{x=a}{\overset{x=b}{\mathsf{A}}} x^n$$

Definition 1: The function $f(x)$ is a point $f(a)$ at $x = a$.

$$f(x) = \underset{x=a}{\overset{x=b}{\mathsf{A}}} x^n$$

Definition 2: An addition operator has a domain that can be an inequality of x.

$$f(x) = \underset{a \leq x}{\overset{x<b}{\mathsf{A}}} x^n$$

Definition 3: An alternating pattern for two functions has addition operators with inequalities.

$$f(x_1) = \underset{x_2=0}{\overset{n}{\mathsf{A}}} \left(\underset{2x_2 \leq x_1}{\overset{x_1 < 2x_2+1}{\mathsf{A}}} g(x_1) + \underset{2x_2+1 \leq x_1}{\overset{x_1 < 2x_2+2}{\mathsf{A}}} h(x_1) \right)$$
$$= \left(\underset{0 \leq x_1}{\overset{x_1<1}{\mathsf{A}}} g(x_1) + \underset{1 \leq x_1}{\overset{x_1<2}{\mathsf{A}}} h(x_1) \right) + \left(\underset{2 \leq x_1}{\overset{3<x_1}{\mathsf{A}}} g(x_1) + \underset{3 \leq x_1}{\overset{x_1<4}{\mathsf{A}}} h(x_1) \right) + \cdots$$

Section 3.2: Counter operator and addition operator theorems

The summation operator is identified as two operators.

Definition 1: A function has terms of $g(x)$.

Theorem 1: The counter operator of $g(x)$ is the simplest operator and is the index counter only. It states that $x = 1$ or $x = 2$ or $x = 3$ etc.

$$f(x) = \underset{x=0}{\overset{x=n}{i}} g(x) = g(0) \text{ or } g(1) \text{ or } g(2) \ldots$$

Theorem 2: The addition and counter operator of $g(x)$ is a function of points.

$$f(x) = \mathsf{A}\left(\overset{x=n}{\underset{x=0}{\imath}} \quad g(x) \right) = g(0) \text{ and } g(1) \text{ and } g(2) \ldots$$

Theorem 3: An operator can be a composite operator of another operator.

Theorem 4: The summation is the addition operator of the counter operator.

$$f(x) = \sum_{x=0}^{x=n} g(x) = \overset{x=n}{\underset{x=0}{\mathsf{A}}} \circ \imath\, g(x)$$

Theorem 5: A continuous counter operator is an or operator for a term $g(x)$.

$$f(x) = \overset{x=b}{\underset{x=a}{\imath_c}} \quad g(x)$$

Definition 1: A point has a position on $g(x)$ that is bounded between $g(a)$ and $g(b)$.

$$f(x) = \overset{x=b}{\underset{x=a}{\imath_c}} \quad g(x)$$

Definition 2: A graph has a width w for the addition operator of the continuous counter operator of a function shifted by a.

$$f(x) = \mathsf{A} \circ \overset{a=w}{\underset{a=0}{\imath_c}} \quad g(x-a)$$

Theorem 6: A list operator is a comma and bracket operation for a function.

$$f(x) = \overset{n}{\underset{x=0}{\mathcal{L}}} g(x) = \{g(0), g(1), \ldots, g(n)\}$$

x	$f(x)$
0	$g(0)$
1	$g(1)$
…	…
n	$g(n)$

Theorem 7: A changing set is a set that has a variable x. The variable $\emptyset$ is the null variable.

$$S(x) - S(\emptyset) = \{f(x), f(x-1), \dots, f(0)\}$$

$$S(x) = \{f(x), f(x-1), \dots, f(0)\}$$

Theorem 8: The addition operator of a counter operator changes open points to closed points.

$$f(x) = \mathrm{A}\,\mathit{i}\, g(x)$$

Theorem 9: A function $f(x)$ that has terms $g_1(x)$ or $g_2(x)$ is one of either for two open lines.

$$f(x) = \mathit{i}\, g_1(x) + \mathit{i}\, g_2(x)$$

Definition 3: A function $f(x)$ that has terms $g_1(x)$ or $g_2(x)$ has one addition operator for one of the two counter operator choices.

$$f(x) = \mathrm{A}\, g_1(x) + \mathit{i}\, g_2(x)$$

Definition 4: A function $f(x)$ that has terms of $g_1(x)$ or $g_2(x)$ has one product or another for a counter operator of a product.

$$f(x) = \mathit{i} \prod g_1(x) + \mathit{i} \prod g_2(x)$$

Definition 5: A function $f(x, y)$ that has terms $g(x, y)$ has a line of the surface that is somewhere between $y = a_2$ and $y = b_2$.

$$f(x, y) = \mathop{\mathit{i}}\limits_{y = a_2}^{y = b_2} \circ \mathop{\mathrm{A}}\limits_{x = a_1}^{x = b_1} g(x, y)$$

Definition 6: A function $f(x, y)$ that has terms $g(x, y)$ has a line on the surface that is somewhere between $x = a_1$ and $x = b_1$.

$$f(x, y) = \mathop{\mathit{i}}\limits_{x = a_1}^{x = b_1} \circ \mathop{\mathrm{A}}\limits_{y = a_2}^{y = b_2} g(x, y)$$

Theorem 10: A function $f(x, y)$ that has terms $g(x, y)$ has a countable sum of x somewhere on a line of y.

$$F(x,y) = \underset{y}{\mathop{i}} \circ \sum_{\Delta x=\frac{1}{k}} g(x,y)$$

Definition 7: A function $f(x,y)$ that has terms $g(x,y)$ has a countable sum of y somewhere on a line of x.

$$F(x,y) = \underset{x}{\mathop{i}} \circ \sum_{\Delta y=\frac{1}{k}} g(x,y)$$

Section 3.3: Addition and counter operators for surface functions

Theorem 1: A master symbol M is for a master operator that controls the composite operators. The master operator controls the counting pattern of the other operators.

$$f(x,y) = \lim_{k\to\infty} \underset{\substack{x=a\\ y=p(x)}}{\overset{x=b}{\mathsf{A}_M}} \circ \underset{\Delta y=\frac{1}{k}}{\mathsf{A}} \circ \underset{\Delta x=\frac{1}{k}}{\mathsf{A}} g(x,y)$$

$$f(x,y) = \underset{\substack{x=a\\ y=p(x)}}{\overset{x=b}{\mathsf{A}}} g(x,y)$$

The function is the continuous line of $g(x,y)$ along the path $p(x)$.

Theorem 2 A surface function $f(x,y)$ has an addition operator of the terms $g(x,y)$ on a path of $y = p(x)$.

$$f(x,y) = \underset{\substack{x=a\\ y=p(x)}}{\overset{x=b}{\mathsf{A}}} g(x,y)$$

Theorem 3: A surface function $f(x,y)$ has a continuous counter operator for a point p along a path $y = p(x)$.

$$f(x,y) = \underset{\substack{x=a\\ y=p(x)}}{\overset{x=b}{\mathop{i}_C}} g(x,y)$$

Theorem 4: A surface function $f(x,y)$ has a point that increases on the x axis along a line y and then increases along a different line of y.

$$f(x,y) = \mathop{\mathrm{i}_c}\limits_{y=0}^{y=a} \left(\mathop{\mathrm{i}_c}\limits_{\substack{x=0 \\ y=0}}^{x=a} g(x,y) \right)$$

Theorem 5: A surface function $f(x,y)$ has a line of x that increases along a line of y and then the line of y increases from zero to a.

$$f(x,y) = \mathop{\mathrm{A}}\limits_{y=0}^{y=a} \left(\mathop{\mathrm{A}}\limits_{\substack{x=a \\ y=0}}^{x=b} g(x,y) \right)$$

Theorem 6: A surface function $f(x,y)$ has a line segment from x equals a to b that forms an area with the other line segment y equals zero to a.

$$f(x,y) = \mathop{\mathrm{A}}\limits_{y=0}^{y=a} \circ \mathop{\mathrm{A}}\limits_{\substack{x=a \\ y=0}}^{x=b} g(x,y)$$

The function is every point in the box region.

Theorem 7: A surface function $f(x,y)$ has a point that has an area that is formed from the line segments of x equals a to b along $y = 0$ to $y = 0$ to $y = a$.

$$f(x,y) = \mathop{\mathrm{i}_c}\limits_{y=0}^{y=a} \circ \mathop{\mathrm{i}_c}\limits_{\substack{x=a \\ y=0}}^{x=b} g(x,y)$$

Definition 1: A surface function $f(x,y)$ has a moving point that is two continuous counter operators of the terms $g(x,y)$.

$$f(x,y) = \mathop{\mathrm{i}_c}\limits_{\Delta y} \circ \mathop{\mathrm{i}_c}\limits_{\Delta x} g(x,y)$$

Theorem 8: A surface function $f(x,y)$ has a list operator that is a table for a pair of x and y. The table of all integer points has two addition operators of the terms $g(x,y)$.

$$\mathcal{L} f(x,y) = \underset{\Delta y = 1}{\mathsf{A}} \circ \underset{\Delta x = 1}{\mathsf{A}} \; g(x,y)$$

(x,y)	1	2	3	4
1	a_1	a_2	a_3	a_4
2	b_1	b_2	b_3	b_4
3	c_1	c_2	c_3	c_4
4	d_1	d_2	c_3	d_4

Section 3.4: Repeatable domains

Addition operators increase the literacy of mathematics for simple problems. A function that has the possibility of more than one point for a value of x is a repeatable function. Repeatable domains are useful for reciprocating functions. A statistical function has several values for one x that is for various categories on $f(x)$.

Theorem 1: A repeatable domain is a set of x that has any number of values for the same point.

$$x\epsilon[a,b]$$

Theorem 2: A range of a repeatable domain has at least two points that have different values in the set $S(x)$.

$$S = \left\{ g\left(\underset{x=a}{\overset{x=c}{\mathsf{A}}} \; x \right), h\left(\underset{x=b}{\overset{x=b}{\mathsf{A}}} \; x \right) \right\}$$

Points can overlap on the range.

Definition 1: The definite function of the set $S(x)$ is $f(x)$ from a to c.

$$f(x) = \underset{x=a}{\overset{x=c}{\mathsf{A}}} \; g(x) + \underset{x=b}{\overset{x=b}{\mathsf{A}}} \; h(x)$$

Definition 2: A function $f(x)$ that has a repeatable domain has two functions $g(x)$ and $h(x)$ with a set of points $S(x)$.

$$f(x) = \underset{x=a}{\overset{x=b}{\mathsf{A}}} \; g(x) + \underset{x=b}{\overset{x=c}{\mathsf{A}}} \; h(x)$$

Section 3.5: Reciprocating functions

Definition 1: A function that has a repeatable domain can have a repeatable range.

Theorem 1: A function has a reciprocating variable from x_1 to x_2 that is the counter operator of the addition operator.

$$x_1 = x, \qquad 0 \le x \le M$$

$$x_2 = M - x, \qquad 0 \le x \le M$$

$$x = \underset{i=1}{\overset{i=2}{\mathrm{i}}} \circ \underset{x=0}{\overset{x=M}{\mathrm{A}}} x_i$$

$$f(x) = f\left(\underset{i=1}{\overset{i=2}{\mathrm{i}}} \circ \underset{x=0}{\overset{x=M}{\mathrm{A}}} x_i \right)$$

Theorem 2: A function has a reciprocating variable of n cycles for x_1 and x_2 that is the double counter operator of the addition operator.

$$x_1 = x, \qquad 0 \le x \le M$$

$$x_2 = M - x, \qquad 0 \le x \le M$$

$$x = \underset{k=1}{\overset{k=n}{\mathrm{i}}} \circ \underset{i=2k-1}{\overset{i=2k}{\mathrm{i}}} \circ \underset{x=0}{\overset{x=M}{\mathrm{A}}} x_i$$

$$f(x) = f\left(\underset{k=1}{\overset{k=n}{\mathrm{i}}} \circ \underset{i=2k-1}{\overset{i=2k}{\mathrm{i}}} \circ \underset{x=0}{\overset{x=M}{\mathrm{A}}} x_i \right)$$

Theorem 3: The inverse function of an alternating variable must have the cycle number for the variable.

$$y = f^{-1}(x)\, for\ k = p$$

Chapter 7: New differentiation

Section 7.1: Horizontal derivative

Definition 1: The slope of a function is the change in y divided by the change in x.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Theorem 1: The derivative is the instantaneous rate of change of the function.

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{(x + \Delta x) - x} = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

The derivative is the slope of a function for an x axis that does not move and to move the x axis without changing values the derivative is said to be the vertical rate of change only.

Theorem 2: The derivative is the vertical rate of change only for a divisor of only delta x and the change to the right is the horizontal derivative.

Definition 2: The variable does not have a function for the denominator of the derivative.

Theorem 3: The derivative has an arbitrary horizontal velocity of $v_h(x)$ that is multiplied by $f'(x)$.

$$v(x) = v_h(x)f'(x)$$

The Leibniz notation is $\frac{dy}{dx}$ for $f'(x)$ and $\frac{dv_h}{dx}$ for $v_h(x)$.

$$v(x) = \frac{dv_h}{dx}\frac{dy}{dx}$$

Definition 3: The rate of change of $f(x)$ at a velocity of zero is zero.

$$f'(x) = \lim_{\Delta x \to 0} 0 \cdot \frac{f(x + \Delta x) - f(x)}{\Delta x} = 0$$

Section 7.2: Assigning a derivative

Theorem 1: A derivative can be assigned to a function because the rate of change can be of an arbitrary function.

$$f_2'(x) = \lim_{\Delta x \to 0} \frac{f_2(x + \Delta x) - f_2(x)}{\Delta x}$$

Theorem 2: The assigned derivative has a horizontal velocity that is solved for from the fact that $v(x) = v_h(x)f_1'(x)$.

$$v_h(x)f_1'(x) = f_2'(x)$$

$$v_h(x) = \frac{f_2'(x)}{f_1'(x)}$$

Theorem 3: A function $f_1(x)$ with an assigned derivative $f_2'(x)$ for $f_1(x)$ has a point with motion on the graph of $f_2'(x)$. The point is increasing horizontally by the horizontal derivative $v_h(x)$.

Section 7.3: Range derivative

Theorem 1: A function has a derivative $f'(x)$ and the rate of change of $f'(x)$ by the range of $f(x)$ is the product of $\pm f'(x)f'(x)$.

Proof: A point has a change of $f'(x)$ and the point has an axis that changes by $f'(x)$.

$$f_R'(x) = \lim_{\Delta x \to 0} \pm \frac{f(x + \Delta x) - f(x)}{\Delta x} \cdot \frac{f(x + \Delta x) - f(x)}{\Delta x} = \pm f'(x)^2$$

Definition 1: A function $f(x)$ that has a derivative $f'(x)$ that is multiplied by another derivative is the product $\pm f_1'(x)f_2'(x)$.

$$f_T'(x) = \pm f_1'(x)f_2'(x)$$

Definition 2: A function that has a derivative has a horizontal derivative times the range derivative.

$$f_R'(x) = \lim_{\Delta x \to 0} \pm \left(\frac{f_1(x + \Delta x) - f_1(x)}{\Delta x} \right)^2 \left(\frac{f_2(x + \Delta x) - f_2(x)}{\Delta x} \right)$$

$$f_R'(x) = \pm f_1'(x)^2 f_2'(x)$$

Theorem 2: A range derivative is returned to the function by the integral of the positive or negative square root of the absolute value of $f_R'(x)$

$$F(x) = \pm \int \sqrt{|f_R'(x)|} dx$$

Theorem 3: All negative derivatives have a sign that must remain for the range derivative.

Theorem 4: All alternating sign derivatives must have the same sign change for the range derivative.

Theorem 5: A function $f_1(x)$ that has a range derivative and a function $f_2(x)$ that has a range derivative has a range change between them of $f_{\Delta R}'(x)$.

$$f_1'(x) = \lim_{\Delta x \to 0} \frac{f_1(x + \Delta x) - f_1(x)}{\Delta x}$$

$$f_2'(x) = \lim_{\Delta x \to 0} \frac{f_2(x + \Delta x) - f_2(x)}{\Delta x}$$

$$f_{1R}'(x) = \pm f_1'(x)^2$$

$$f_{2R}'(x) = \pm f_2'(x)^2$$

$$f_{\Delta R}'(x) = \pm |f_2'(x)^2 - f_1'(x)^2|$$

Theorem 6: A function $f_1(x)$ that has a horizontal derivative $v_{h_1}(x)$ and a function $f_2(x)$ that has a horizontal derivative $v_{h_2}(x)$ has a change in location of $v_{\Delta h}(x)$.

$$v_{h_1}(x) = g_1'(x)$$

$$v_{h_2}(x) = g_2'(x)$$

$$v_{\Delta h}(x) = g_2'(x) - g_1'(x)$$

Theorem 7: A function $f_1(x)$ that has a horizontal derivative and range derivative and a function $f_2(x)$ that has a horizontal and range derivative has a total change of $f_{\Delta T}'(x)$.

$$v_{h1}(x) = g_1'(x)$$

$$v_{h_2}(x) = g_2'(x)$$

$$f_{1R}'(x) = \pm f_1'(x)^2$$

$$f_{2R}'(x) = \pm f_2'(x)^2$$

$$f'_{\Delta T}(x) = \pm\left|v_{h_2}(x) f_2'(x)^2 - v_{h_1}(x) f_1'(x)^2\right|$$

Chapter 8: The mean

Section 8.1: The mean

Definition 1: The limit of a counter step is taken first before the arithmetic.

$$y = \lim_{k\to\infty} \sum_{\Delta x=\frac{1}{k}} f(x)$$

A substitution for counter step is a value of k such as 200 or 400.

Theorem 1: A function $f(x)$ has a limit mean of k_1 for the mean counter and k_2 for the change.

$$\mu(x) = \lim_{\substack{k_2\to\infty \\ k_1\to\infty}} \frac{1}{k_2} \sum_{\Delta x=\frac{1}{k_1}} f(x)$$

Theorem 2: The mean of all points for a counter step of $\Delta x = \frac{1}{k_1}$ is equal to the integration coefficient times x^n from zero to a for power functions.

$$\mu(a) = \lim_{\substack{k_1\to\infty \\ k_2\to\infty}} \frac{1}{k_2} \sum_{\substack{\Delta x=\frac{1}{k_1} \\ x=0}}^{x=a} x^n = \frac{1}{n+1} a^n$$

The mean between two points that does not begin with zero has a mean that can be calculated.

$$\mu = \lim_{\substack{k_1\to\infty \\ k_2\to\infty}} \frac{1}{k_2} \sum_{\substack{\Delta x=\frac{1}{k_1} \\ x=a}}^{x=b} x^n$$

Proof 1: The difference in two means from zero is not the correct solution.

$$\mu(b) - \mu(a) \neq \lim_{\substack{k_1\to\infty \\ k_2\to\infty}} \frac{1}{k_2} \sum_{\substack{\Delta x=\frac{1}{k_1} \\ x=0}}^{x=b} x^n - \lim_{\substack{k_1\to\infty \\ k_2\to\infty}} \frac{1}{k_2} \sum_{\substack{\Delta x=\frac{1}{k_1} \\ x=0}}^{x=b} x^n$$

The integral solution to the mean for any two points is easy for power functions.

$$\mu = \frac{1}{b-a} \int_a^b x^n \, dx = \frac{1}{b-a} \frac{1}{n+1} \left(b^{n+1} - a^{n+1}\right)$$

Proof 2: If the integral for a power function has an integration coefficient of $\frac{1}{n+1}$ than the integral can be separated into a mean times x. The sample points are x_i and $*$ implies equally spaced points.

$$\int f(x)dx = \lim_{k\to\infty} \sum_{i=1}^{k} \Delta x f(x_i^*)$$

If $\Delta x = \frac{b-a}{k_1}$ than $b-a$ is equal to a difference of x and k_1 and k_2 is for the count of points.

$$\int_a^b f(x)dx = \lim_{\substack{k_1\to\infty \\ k_2\to\infty}} \frac{b-a}{k_2} \sum_{\substack{\Delta x=\frac{1}{k_1} \\ x=a}}^{x=b} f(x)$$

The difference of $b\mu(b) - a\mu(a)$ is equal to the product across a and b. The function has a mean from a to b that is μ_1 and a mean function from zero that is $\mu_2(x)$.

$$\int_a^b f(x)dx = \lim_{\substack{k_1\to\infty \\ k_2\to\infty}} \frac{b-a}{k_2} \sum_{\substack{\Delta x=\frac{1}{k_1} \\ x=a}}^{x=b} f(x) = (b-a)\mu_1$$

$$\int_a^b f(x)dx = b\mu_2(b) - a\mu_2(a)$$

The pure mean is $\mu(x)$.

$$\mu(x) = \lim_{\substack{k_1\to\infty \\ k_2\to\infty}} \frac{1}{k_2} \sum_{\Delta x=\frac{1}{k_1}} f(x)$$

Definition 3: A function that has a mean that begins at zero is $\mu(a)$.

$$\mu(a) = \lim_{\substack{k_1\to\infty \\ k_2\to\infty}} \frac{1}{k_2} \sum_{\substack{\Delta x=\frac{1}{k_1} \\ x=0}}^{x=a} f(x)$$

Definition 4: A function that has a mean across a distance that does not begin at zero is μ.

$$\mu = \lim_{\substack{k_1\to\infty \\ k_2\to\infty}} \frac{1}{k_2} \sum_{\substack{\Delta x=\frac{1}{k_1} \\ x=a}}^{x=b} f(x)$$

Definition 5: The mean of one point is the point.

$$\mu = \lim_{\substack{k_1\to\infty \\ k_2\to\infty}} \frac{1}{k_2} \sum_{\substack{\Delta x=\frac{1}{k_1} \\ x=a}}^{x=a} f(x) = \frac{1}{1} f(a) = f(a)$$

Definition 6: The mean derivative is the mean of $f'(x)$.

$$\mu(x) = \lim_{\substack{k_1 \to \infty \\ k_2 \to \infty}} \frac{1}{k_2} \sum_{\Delta x = \frac{1}{k_1}} f'(x)$$

Definition 7: The midpoint of the range is the maximum minus the minimum of $f(x)$ divided by two.

Definition 8: A mean with a counter step k_1 of 200 has approximately one decimal place correct for power functions in a small range of x.

Definition 9: A mean with a counter step k_1 of 400 has approximately two decimal places correct for power functions in a small range of x.

The mean for x was calculated with a counter step k_1 of 400 and the mean for x was confirmed to be of the form $\frac{1}{2}x$.

Value	Number of Rows	Calculated Mean	Mean Equation $\frac{1}{2}x$
1	400	0.50125	0.5
2	800	1.00125	1
3	1200	1.50125	1.5
4	1600	2.00125	2

The mean for x^2 was calculated with a counter step k_1 of 400 and the mean was confirmed to be of the form $\frac{1}{n+1}x^n$.

Value	Number of Rows	Calculated Mean	Mean Equation $\frac{1}{3}x^2$
1	400	0.3345	0.333
2	800	1.3358	1.333
3	1200	3.0037	3
4	1600	5.3383	5.333

The mean for x^3 was calculated with a counter step k_1 of 400 and the mean was confirmed to be of the form $\frac{1}{n+1}x^n$.

Value	Number of Rows	Calculated Mean	Mean Equation $\frac{1}{4}x^3$
1	400	0.2512	0.25
2	800	2.0050	2
3	1200	6.7612	6.75
4	1600	16.0200	16

The indefinite integral is the mean times x.

$$\int f(x)dx = x\mu(x)$$

Theorem 3: The mean of a function $f(x)$ from a to b is one divided by $b-a$ times the definite integral.

$$\mu = \lim_{\substack{k_1\to\infty \\ k_2\to\infty}} \frac{1}{k_2} \sum_{\substack{\Delta x=\frac{1}{k_1} \\ x=a}}^{x=b} f(x) = \frac{1}{b-a}\int_a^b f(x)\,dx$$

Definition 10: The mean of a radical function that begins at zero is the integral coefficient times the function.

$$\mu(x) = \lim_{\substack{k_1\to\infty \\ k_2\to\infty}} \frac{1}{k_2} \sum_{\Delta x=\frac{1}{k_1}} x^{\frac{1}{n}} = \frac{x^{\frac{1}{n}}}{1+\frac{1}{n}} = \frac{n}{n+1}x^{\frac{1}{n}}$$

The mean between two points a and b is one divided by the difference of b and a times the integral of $x^{\frac{1}{n}}$ from a to b.

$$\mu = \lim_{\substack{k_1\to\infty \\ k_2\to\infty}} \frac{1}{k_2} \sum_{\substack{\Delta x=\frac{1}{k_1} \\ x=a}}^{x=b} x^{\frac{1}{n}} = \frac{1}{b-a}\int_a^b x^{\frac{1}{n}}\,dx$$

Definition 11: The reciprocal function has a mean that is the absolute value of the natural logarithm divided by the difference of bounds.

$$\mu = \lim_{\substack{k_1\to\infty \\ k_2\to\infty}} \frac{1}{k_2} \sum_{\substack{\Delta x=\frac{1}{k_1} \\ x=a}}^{x=b} \frac{1}{x} = \frac{1}{b-a}\int_a^b \frac{1}{x}dx = \frac{1}{b-a}(\ln|b| - \ln|a|)$$

Proof 3: The mean of the reciprocal function is the integral of the reciprocal function divided by x.

$$\int \frac{1}{x}dx = x\mu(x)$$

$$\mu(x) = \frac{1}{x}\int \frac{1}{x}dx$$

The left side x must be evaluated for the bounds.

$$\mu = \frac{1}{b-a}\int_a^b \frac{1}{x}dx$$

$$\mu = \frac{1}{b-a}(\ln|b| - \ln|a|)$$

Definition 12: The mean of the natural logarithm is the natural logarithm minus one.

$$\mu = \frac{1}{b-a}\int_a^b \ln|x|\,dx = \frac{1}{b-a}\big((b\ln|b| - b) - (a\ln|a| - a)\big)$$

Definition 13: A function $f(x)$ has a derivative that has a mean that has a range derivative.

$$f(x) = \int f'(x)dx$$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$\mu(x) = \lim_{\substack{k_1 \to \infty \\ k_2 \to \infty}} \frac{1}{k_2} \sum_{\Delta x = \frac{1}{k_1}} f'(x)$$

$$\mu_R'(x) = \pm\mu(x)^2$$

Section 8.2: Mean experiment for the base function

Definition 1: A base function mean with a counter step k_1 of 400 is accurate to approximately one decimal place for a small range of x. For small functions the accuracy is to two decimal places.

Theorem 1: The mean of the base function is the definite integral divided by the difference in the two bounds.

$$\mu = \lim_{\substack{k_1 \to \infty \\ k_2 \to \infty}} \frac{1}{k_2} \sum_{\substack{\Delta x = \frac{1}{k_1} \\ x=a}}^{c} b^x = \frac{1}{c-a}\int_a^c b^x dx$$

$$\mu = \frac{1}{(c-a)\ln b}(b^c - b^a)$$

The exponential function has a calculated integral of the mean times the distance.

$$\int_0^a e^x = \frac{a}{a\ln e}(e^a - e^0) = e^a - 1$$

The table for the exponent power was calculated with a counter step k_1 of 1000.

Function	Value	Mean	Calculated Integral	Integral
EXP(0)	1.0000	1.0000	0.0000	0.0000
EXP(0.25)	1.2840	1.1407	0.2852	0.2840
EXP(0.5)	1.6487	1.3001	0.6500	0.6487
EXP(0.75)	2.1170	1.4914	1.1186	1.1170
EXP(1)	2.7183	1.7201	1.7201	1.7183
EXP(1.25)	3.4903	1.9941	2.4926	2.4903
EXP(1.5)	4.4817	2.3230	3.4844	3.4817
EXP(1.75)	5.7546	2.7188	4.7580	4.7546

EXP(2)	7.3891	3.1966	6.3933	6.3891
EXP(2.25)	9.4877	3.7747	8.4930	8.4877
EXP(2.5)	12.1825	4.4756	11.1891	11.1825
EXP(2.75)	15.6426	5.3276	14.6510	14.6426
EXP(3)	20.0855	6.3654	19.0961	19.0855
EXP(3.25)	25.7903	7.6319	22.8958	24.7903
EXP(3.5)	33.1155	9.1807	32.1325	32.1155
EXP(3.75)	42.5211	11.0781	41.5428	41.5211
EXP(4)	54.5982	13.4065	53.6260	53.5982
EXP(4.25)	70.1054	16.2685	69.1410	69.1054
EXP(4.5)	90.0171	19.7917	89.0626	89.0171
EXP(4.75)	115.5843	24.1353	114.6426	114.5843
EXP(5)	148.4132	29.4976	147.4879	147.4132

$$\int_0^a e^{-x} = -\frac{a}{a\ln e}(e^{-a} - e^0) = -(e^{-a} - 1) = 1 - e^{-a}$$

The table for the exponent with a negative power was calculated with a counter step k_1 of 1000.

Function	Value	Mean	Calculated Integral	Integral
EXP(0)	1.0000	1.0000	0.0000	0.0000
EXP(-0.25)	0.7788	0.8884	0.2221	0.2212
EXP(-0.5)	0.6065	0.7885	0.3943	0.3935
EXP(-0.75)	0.4724	0.7045	0.5284	0.5276
EXP(-1)	0.3679	0.6328	0.6328	0.6321
EXP(-1.25)	0.2865	0.5713	0.7141	0.7135
EXP(-1.5)	0.2231	0.5183	0.7775	0.7769
EXP(-1.75)	0.1738	0.4725	0.8268	0.8262
EXP(-2)	0.1353	0.4326	0.8652	0.8647
EXP(-2.25)	0.1054	0.3978	0.8952	0.8946
EXP(-2.5)	0.0821	0.3674	0.9185	0.9179
EXP(-2.75)	0.0639	0.3406	0.9366	0.9361
EXP(-3)	0.0498	0.3169	0.9507	0.9502
EXP(-3.25)	0.0388	0.2959	0.8878	0.9612

EXP(-3.5)	0.0302	0.2772	0.9703	0.9698
EXP(-3.75)	0.0235	0.2605	0.9770	0.9765
EXP(-4)	0.0183	0.2455	0.9822	0.9817
EXP(-4.25)	0.0143	0.2321	0.9862	0.9857
EXP(-4.5)	0.0111	0.2199	0.9894	0.9889
EXP(-4.75)	0.0087	0.2088	0.9919	0.9913
EXP(-5)	0.0067	0.1988	0.9938	0.9933

Section 8.3: The mean of trigonometric functions

The mean of sine and cosine begins with no rows for the first entry and 100 rows for each $\frac{\pi}{8}$ entry.

Radians	Mean of Sine	Mean of Cosine
0.0000	0.0000	1.0000
0.3927	0.1958	0.9841
0.7854	0.3747	0.9046
1.1781	0.5255	0.7865
1.5708	0.6379	0.6379
1.9635	0.7051	0.4711
2.3562	0.7251	0.3003
2.7489	0.7001	0.1393
3.1416	0.6366	0.0000

Theorem 1: The mean of sine is equal to $\frac{1}{b-a}$ times the definite integral of sine.

$$\mu = \lim_{\substack{k_1 \to \infty \\ k_2 \to \infty}} \frac{1}{k_2} \sum_{\substack{\Delta x = \frac{1}{k_1} \\ x=a}}^{x=b} \sin x = \frac{1}{b-a} \int_a^b \sin x \, dx$$

$$\mu = -\frac{1}{b-a}(\cos b - \cos a)$$

$$\mu = \frac{1}{b-a}(\cos a - \cos b)$$

Theorem 2: The mean of cosine is equal to $\frac{1}{b-a}$ times the definite integral of cosine from a to b.

$$\mu = \lim_{\substack{k_1\to\infty \\ k_2\to\infty}} \frac{1}{k_2} \sum_{\substack{\Delta x=\frac{1}{k_1} \\ x=a}}^{x=b} \cos x = \frac{1}{b-a}\int_a^b \cos x\,dx$$

$$\mu = \frac{1}{b-a}(\sin b - \sin a)$$

Theorem 3: The mean of tangent is equal to $\frac{1}{b-a}$ times the definite integral of tangent.

$$\mu = \lim_{\substack{k_1\to\infty \\ k_2\to\infty}} \frac{1}{k_2} \sum_{\substack{\Delta x=\frac{1}{k_1} \\ x=a}}^{x=b} \tan x = \frac{1}{b-a}\int_a^b \tan x\,dx$$

$$\mu = -\frac{1}{b-a}(\ln|\cos b| - \ln|\cos a|)$$

$$\mu = -\frac{1}{b-a}\ln\left(\frac{|cos\, b|}{|\cos a|}\right)$$

$$\mu = \frac{1}{b-a}\ln\left(\frac{|\cos a|}{|\cos b|}\right)$$

Section 8.4: The mean of a polynomial

Theorem 1: The mean of a polynomial is equal to the mean of each term in the polynomial.

Theorem 2: The coefficient function $c(x)$ for polynomials has a definition of the function divided by the mean of each term in the polynomial.

$$\frac{1}{c(x)}f(x) = \mu_1(x) + \mu_2(x) + \cdots$$

$$c(x) = \frac{f(x)}{\mu_1(x) + \mu_2(x) + \cdots}$$

The function $f(x) = x + x^2$ has a mean with a coefficient function $c(x)$.

$$\mu(x) = \lim_{\substack{k_1\to\infty \\ k_2\to\infty}} \frac{1}{k_2} \sum_{\Delta x=\frac{1}{k_1}} (x + x^2) = \frac{1}{c(x)}(x + x^2)$$

The coefficient function $c(x)$ is $c(x) = \frac{1+x}{\frac{1}{2}+\frac{1}{3}x}$.

$$\frac{1}{c(x)}(x + x^2) = \frac{1}{2}x + \frac{1}{3}x^2$$

$$\frac{1}{c(x)} = \frac{\frac{1}{2}x + \frac{1}{3}x^2}{x + x^2}$$

$$\frac{1}{c(x)} = \frac{\frac{1}{2} + \frac{1}{3}x}{1 + x}$$

$$c(x) = \frac{1 + x}{\frac{1}{2}x + \frac{1}{3}x}$$

x	Mean	$f(x)$	Coefficient	Mean of each term
1	0.8383	2	2.3857	0.8333
2	2.3408	6	2.5632	2.3333
3	4.5100	12	2.6608	4.5000
4	7.3458	20	2.7226	7.3333
5	10.8483	30	2.7654	10.8333
6	15.0175	42	2.7967	15.0000
7	19.8533	56	2.8207	19.8333
8	25.3558	72	2.8396	25.3333
9	31.5250	90	2.8549	31.5000
10	38.3608	110	2.8675	38.3333

The table has a counter step of $\Delta x = \frac{1}{200}$.

Section 8.5: Convergence for means

Definition 1: A power function $f(x)$ has a mean coefficient $c(x)$ that has a limit function to the convergent value.

$$f(x) = x^n$$

$$\mu(x) = \lim_{\substack{k_1 \to \infty \\ k_2 \to \infty}} \sum_{\Delta x = \frac{1}{k_1}} f(x) = \frac{f(x)}{c(x)}$$

$$c_l(x) = \lim_{x \to \infty} c(x)$$

$$c_l(x) = n + 1$$

Theorem 1: A radial function that has a constant in the radical has a mean coefficient $c(x)$ that has a limit function $c_l(x)$ to the convergent value.

$$f(x) = \sqrt[n]{C + x^m}$$

$$\mu(x) = \lim_{\substack{k_1 \to \infty \\ k_2 \to \infty}} \frac{1}{k_2} \sum_{\Delta x = \frac{1}{k_1}} f(x) = \frac{f(x)}{c(x)}$$

$$c_l(x) = \lim_{x \to \infty} c(x)$$

$$c_l(x) = \frac{n}{m} + 1$$

Definition 2: A base function $f(x)$ has a mean coefficient function $c(x)$ that has a limit function $c_l(x)$ to the convergent value.

$$f(x) = b^x$$

$$\mu(x) = \lim_{\substack{k_1 \to \infty \\ k_2 \to \infty}} \frac{1}{k_2} \sum_{\Delta x = \frac{1}{k_1}} f(x) = \frac{f(x)}{c(x)}$$

$$c_l(x) = \lim_{x \to \infty} c(x)$$

$$c_l(x) = x$$

Definition 3: A radical function that has a constant C and a base b^x has a mean coefficient function $c_l(x)$ that has a limit to the convergent value.

$$f(x) = \sqrt[n]{C + b^x}$$

$$\mu(x) = \lim_{\substack{k_1 \to \infty \\ k_2 \to \infty}} \frac{1}{k_2} \sum_{\Delta x = \frac{1}{k_1}} f(x) = \frac{f(x)}{c(x)}$$

$$c_l(x) = \lim_{x \to \infty} c(x)$$

The base limit coefficient function is x.

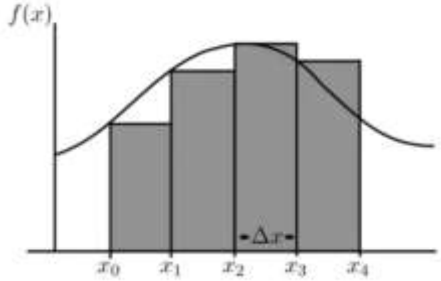
$$c_l(x) = \lim_{x \to \infty} c(x) = x$$

Chapter 9: Integration for area and volume

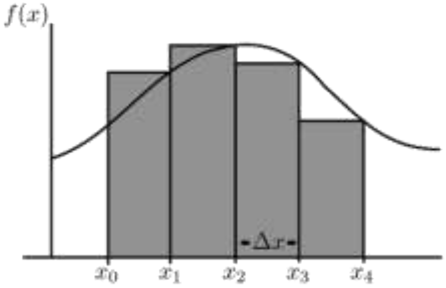
Section 9.1: Integration

There is no longer a limit Riemann sum with the new integral theorem. The new integration also allows for discontinuous sets to be integrated.

$$\int f(x)dx = \lim_{k\to\infty} \sum_{i=1}^{k} \Delta x f(x_i^*)$$



The upper Riemann sum was weaker than the lower Riemann sum because the initial point of the function was not included.



Theorem 1: The area of a function is the integral and is the sum.

$$A(x) = \int f(x)dx = x\mu(x)$$

Theorem 2: The derivative of a function is returned to the function by the integral.

$$\int f'(x)dx = \lim_{\substack{k_1\to\infty \\ k_2\to\infty}} \frac{x}{k_2} \sum_{\Delta x=\frac{1}{k_1}} f'(x) = x\mu(x)$$

Definition 1: A function must be continuous from a to b for the integral to be a continuous function.

Definition 2: The definite integral is the name for an integral with two evaluated bounds that are a closed domain $[a,b]$.

$$\int_a^b f'(x)dx = F(b) - F(a)$$

Definition 3: The integral of a constant times a variable function is equal to the constant times the integral of the variable function.

$$\int kf(x)dx = k\int f(x)\,dx$$

Definition 4: The integral of a point is equal to zero.

$$\int_a^a f(x)\,dx = 0$$

Definition 5: The integral from a to b of $f(x)$ is also equal to the negative integral from b to a.

$$\int_a^b f(x)\,dx = -\int_b^a f(x)$$

Definition 6: The integral of $f(x)$ from a to c is equal to two integrals from a to b and b to c.

$$\int_a^c f(x)dx = \int_a^b f(x)dx + \int_b^c f(x)dx$$

Definition 7: The integral of two functions is equal to the sum or subtraction of two integrals.

$$\int f(x) \pm g(x)dx = \int f(x)dx \pm \int g(x)dx$$

Definition 8: The integral of $f(t)$ is the integral of the time variable and the placement of f is at the position $f(x)$.

$$\int_0^x f'(t)dt = f(x)$$

Definition 9: The derivative of $f(x)$ has a substitution of t to change the variable back into the time that counted the derivative.

$$\frac{d}{dx}f(x) = f'(x), \qquad x = t$$

$$\frac{d}{dt}f(t) = f'(t)$$

Definition 10: A function completed at different velocity is a new function $f_2(x)$. The total of $f(x)$ must be equal to the total of $f_2(x)$.

$$f_2(x) = \int v(x)dx$$

Definition 11: The integral of $v_h(x)$ is the total distance across the x axis.

$$d(x) = \int v_h(x)\,dx$$

Theorem 3: A sum of $f(x)$ that is completed at a different velocity is the sum $F_2(x)$ that has a total equal to first total $F_1(x)$.

$$F_2(x) = \int v_h(x)f(x)dx$$

Theorem 4: A power scale has a sum that is the integral.

$$S(x) = \int s(x)dx$$

Definition 12: A sum of a set is the sum of each number in the set.

$$S = \{a, b, \dots, z\}$$

$$y = \sum S = a + b + \cdots + z$$

Theorem 5: The integral of a set of non-continuous points is the mean of all the points times x.

$$S = \{f(x_1), f(x_2), \dots, f(x_n)\}$$

$$\mu(x) = \lim_{\substack{k_1\to\infty \\ k_2\to\infty}} \frac{1}{k_2} \sum f(x)$$

$$F(x) = \int S dx = x\mu(x)$$

$$F(b) - F(a) = \int_a^b S dx = b\mu(b) - a\mu(a)$$

Definition 13: A statistical set of S that has a large k has an integral that is $x\mu(x)$.

$$F(x) = \int S dx = x\mu(x)$$

$$F(b) - F(a) = \int_a^b S dx = b\mu(b) - a\mu(a)$$

Definition 14: A piecewise set has piecewise integrals.

Definition 15: A point function $f(x)$ has a variable x counted by $\Delta x = \frac{1}{k}$.

Theorem 6: A point function of $\Delta x = \frac{1}{k}$ has an integral that is a discontinuos sum of $f(x)$.

$$F(x) = \int f(x_{nk})dx$$

$$F(n) - F(0) = \int_0^n f(x_{nk})dx$$

Theorem 7: A function of all non-continuous points has a sum without an area theorem.

$$F(x) = \int f(x)dx$$

A point integral is useful for bounded sums that are already known. An integral of only a few points can be larger than the limit integral and the point integral is the best computation tool.

Theorem 8: A difficult function has an integral that is of the form $\frac{x}{c(x)}f(x)$.

$$\int f(x)\,dx = \lim_{\substack{k_1\to\infty \\ k_2\to\infty}} \frac{x}{k_2} \sum_{\Delta x=\frac{1}{k_1}} f(x) = \frac{x}{c(x)}f(x)$$

Definition 16: The integral of the n^{th} radical of a polynomial $p(x)$ divided by a polynomial function $q(x)$ is $\frac{x}{c(x)}f(x)$. The coefficient function $c(x)$ may have a model that is easy to discover in a computer using the mean.

$$f(x) = \sqrt[n]{\frac{p(x)}{q(x)}}$$

$$\int \sqrt[n]{\frac{p(x)}{q(x)}}dx = \frac{x}{c(x)}\sqrt[n]{\frac{p(x)}{q(x)}}$$

Theorem 9: The double integral returns the function of $f''(x)$ to $f(x)$.

$$\iint f''(x)dxdx = \lim_{\substack{k_3\to\infty \\ k_4\to\infty}} \frac{x}{k_4} \sum_{\Delta x=\frac{1}{k_3}} \lim_{\substack{k_1\to\infty \\ k_2\to\infty}} \frac{x}{k_2} \sum_{\Delta x=\frac{1}{k_1}} f''(x) = f(x)$$

The double integral is false for volumes because one mean must be removed.

Theorem 10: The volume of a function is x^2 times the mean of $f(x)$ and has an evaluation of $V(b) - V(a)$.

$$V(x) = x^2\mu(x)$$

$$V(b) - V(a) = b^2\mu(b) - a^2\mu(a)$$

Definition 19: The mean of the function must be taken to know the height for the area.

Definition 20: The mean of the area is not used because there is no unknown quantities.

Contradiction 1: The double integral is not the volume because the mean of the first area is a new height or a smaller area.

$$A(x) = x\mu_1(x)$$

$$V(x) \neq x\mu_2\big(x\mu_1(x)\big)$$

Theorem 13: A function $f(x)$ has a sum that has a mean of μ.

$$F(x) = \int f(x)dx$$

$$\mu(x) = \lim_{\substack{k_1\to\infty\\ k_2\to\infty}} \frac{1}{k_2} \sum_{\Delta x=\frac{1}{k_1}} \int f(x)dx$$

$$\mu = \lim_{\substack{k_1\to\infty\\ k_2\to\infty}} \frac{1}{k_2} \sum_{\substack{\Delta x=\frac{1}{k_1}\\ x=a}}^{x=b} \int f(x)dx = \frac{1}{b-a}\int_a^b \int f(x)dxdx$$

Theorem 14: A surface function $f(x,y)$ has a volume that is the sum for each point of the function.

$$F(x,y) = \lim_{\substack{k_1\to\infty\\ k_2\to\infty}} \frac{xy}{k_2} \underset{\Delta y = \frac{1}{k_1}}{\mathrm{A}} \circ \sum_{\Delta x=\frac{1}{k_1}} f(x,y)$$